

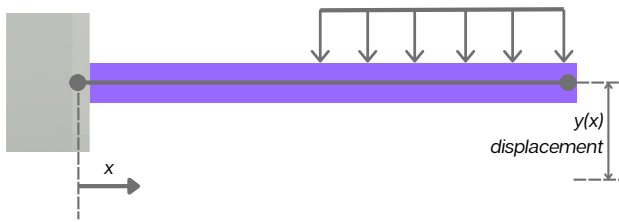
Deflection of Beams

General Equation for Beam Deflection

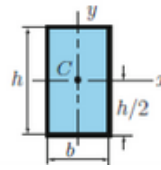
$$\delta = \frac{PL}{EI}$$

P = Load (N)
 L = Beam Length (m)
 E = Modulus of Elasticity (Pa)
 I = Area Moment of Inertia

where E refers to the stiffness of material, and I defines the resistance of a cross-section to bending.



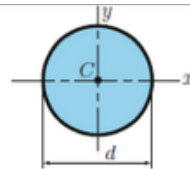
Beam Cross-Section Area Inertia Equations



$$A = bh$$

$$I_{xx} = \frac{bh^3}{12} \quad I_C = \frac{bh}{12}(b^2 + h^2)$$

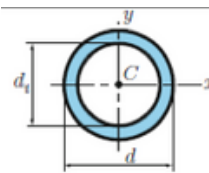
$$I_{yy} = \frac{b^3h}{12}$$



$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

$$I_C = \frac{\pi d^4}{32}$$

$$A = \frac{\pi d^2}{4}$$



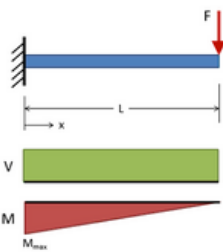
$$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_i^4)$$

$$A = \frac{\pi}{4}(d^2 - d_i^2)$$

$$I_C = \frac{\pi}{32}(d^4 - d_i^4)$$

Common Deflection Equations

Cantilever, End Load

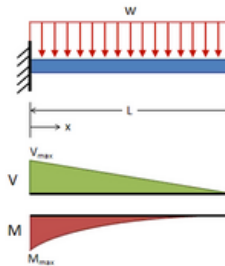


Deflection:

$$\delta = -\frac{Fx^2}{6EI}(3L - x)$$

$$\delta_{\max} = \frac{FL^3}{3EI} \quad @x = L$$

Cantilever, Uniform Distributed Load



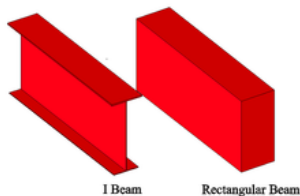
Deflection:

$$\delta = -\frac{wx^2}{24EI}(6L^2 - 4Lx + x^2)$$

$$\delta_{\max} = \frac{wL^4}{8EI} \quad @x = L$$

[Additional Beam Deflection Tables](#)

I-Beam Cross-Section



- I-Beam cross-section optimizes for best strength-to-weight ratio
- Flange (horizontal) resists bending
- Web (vertical) resists shear stresses
- Mol calculated using Parallel Axis Theorem:

$$I = \bar{I} + Ad^2$$

Major Type of Stresses

1. Compression
2. Tension
3. Bending
4. Shear
5. Torsion
6. Fatigue

Stress Equations

Normal (Axial) Stress

$$\sigma = \frac{P}{A}$$

- P = Axial Force
- A = Cross-Sectional Area

*Commonly compression or tension

Shear Stress

$$\tau = \frac{QV}{It}$$

- Q = (Area above point) x (dist. from axis to centroid of area)
- V = Shear force
- I = Mol about neutral axis
- t = Thickness at point

Torsional Stress

$$\tau = \frac{Tr}{J}$$

- T = Applied Torque
- r = Radius of material
- J = Polar moment of inertia

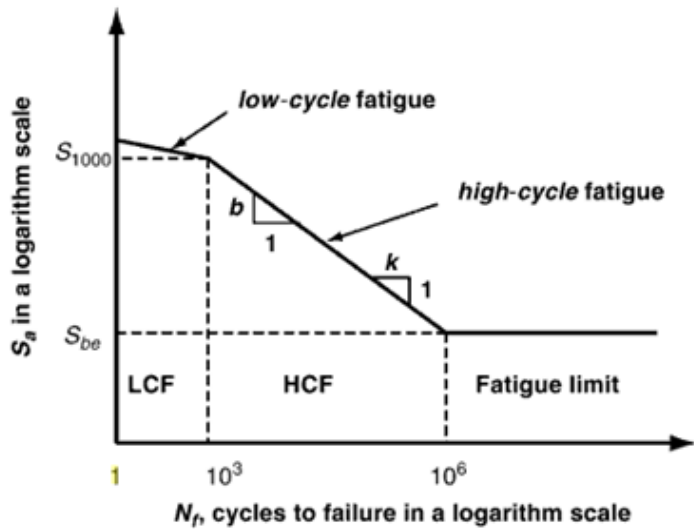
Bending Moment Stress

$$\sigma_b = \frac{My}{I}$$

- M = bending moment
- y = Vertical distance away from neutral axis
- I = Mol about neutral axis

Mechanics of Materials

Fatigue Life & S-N Curves



Curve Definition

Plot of the magnitude of an alternating stress versus the number of cycles to failure for a given material.

Low Cycle Fatigue

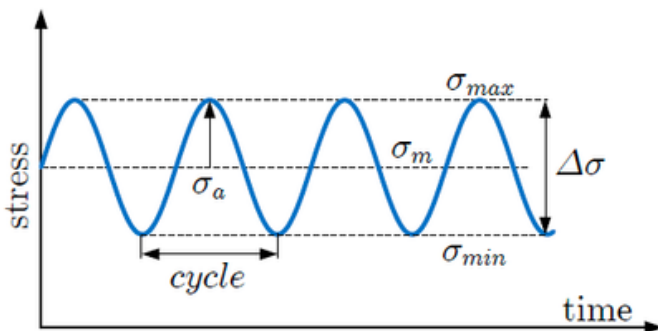
High stresses, small number of cycles until failure (plastic deformation).

High Cycle Fatigue

Low Stresses, large # of Cycles (elastic deformation).

Fatigue Limit

Stress range below where crack growth does not occur and material exhibits infinite life under cyclic stress.

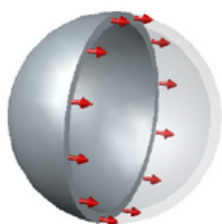


Stages of Fatigue Failure

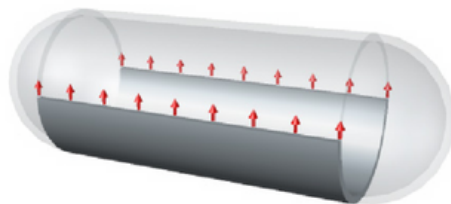
- 1) Crack Formation (occurs at free surfaces + stress concentrations)
- 2) Crack Growth
- 3) Fracture

Fatigue (endurance) limit is applicable only to ferrous (iron-based) metals.

Pressure Vessels



$$\sigma_L = \frac{Pr}{2t}$$



$$\sigma_H = \frac{Pr}{t}$$

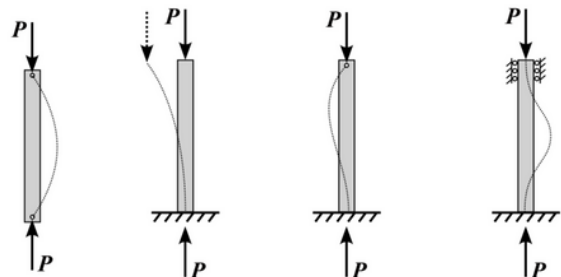
- Hoop stress is always higher
- r = Cross-Sectional Radius
- t = Wall Thickness
- P = Internal - External Pressure

Buckling

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

- E = Young's Modulus
- L = Beam Length
- K = Effective Length Factor

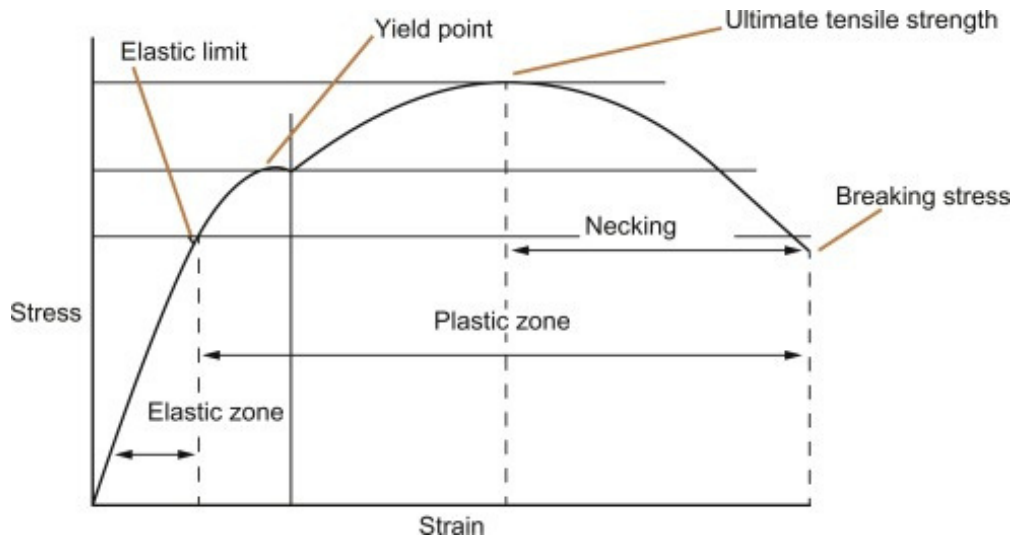
- I = Smallest moment of inertia (also indicates direction of buckling)



- Pin-Pin: $K = 1$ Fixed-Free: $K = 2$ Fixed-Pin: $K = 0.7$ Fixed-Fixed: $K = 0.5$

Materials

Labeled Stress-Strain Curve



Stress Strain Curve Definitions

Equations

Stress - Force applied over cross-sectional area
Strain - Deformation by material relative to original length

$$\epsilon = \frac{\Delta L}{L}$$

Stress-Strain Curve Regions

Elastic Zone

Material deforms linearly in response to applied load. When load removed, material returns to original shape

Plastic Zone - Material deforms permanently

Necking

Tensile deformation where large amount of strain is localized in small region of material

Definitions:

Modulus of Elasticity (Young's Modulus)

Measures a material's capacity to deform elastically under stress

Yield Point

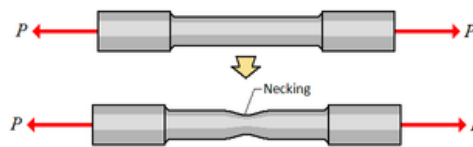
Indicates the limit of elastic behavior and the beginning of plastic behavior.

Ultimate Tensile Strength

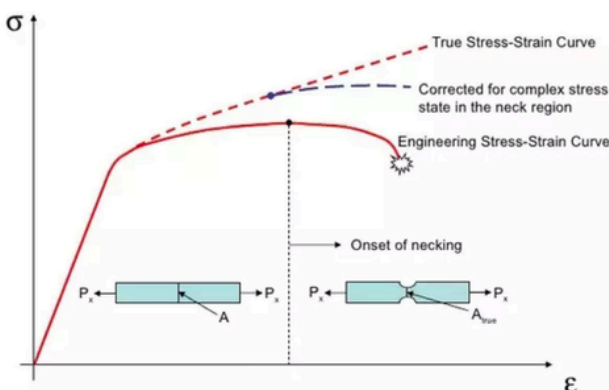
Maximum stress that a material can withstand while being pulled before breaking.

Breaking/Fracture Stress

Stress where material ruptures/breaks

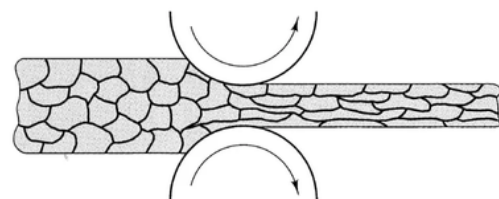


True vs. Engineering Stress-Strain



Strain Hardening

Strengthening of a material by plastic deformation. Occurs because of dislocation movements and dislocation generation within the crystal structure of the material.



Cold Working Diagram

Material Types

Metals

Metallic bonding, tough, heavy, conductive, and medium-high melting points

Ferrous: (Iron, Magnetic, Corrosive) cast iron, high carbon steels, stainless steels

Ex. 316 Stainless Steel (medical equip., engine parts, etc.)

- Tensile YS: 230 MPa
- Young's Modulus: 200 GPa
- Density: 7.9 g/cm³
- Melting Point: 1380 C



Non-ferrous: aluminum, copper, brass

Ex. 7075 Aluminum (aircraft wings, fuselages, etc.)

- Tensile YS: 480 MPa
- Young's Modulus: 70 GPa
- Density: 3.0 g/cm³
- Melting Point: 480 C



Ceramics

Covalent and ionic bonding, brittle, heavy, low conductivity, and high melting points

Traditional: bricks, concrete, glass

Ex. High-Silica Glass (lab glassware, heat-resistance tiles, etc.)

- Tensile YS: 167 MPa
- Young's Modulus: 78 GPa
- Density: 2.4 g/cm³
- Melting Point: 1218 C



Advanced: Alumina, Silicon Carbide

Ex. Silicon Nitride (bearings, cutting tools, etc.)

- Tensile YS: 525 MPa
- Young's Modulus: 280 GPa
- Density: 3.4 g/cm³
- Melting Point: 2495 C



Plastics

Covalent bonding, low electrical conductivity, low melting points, brittle/hard/elastic

Thermoplastics: (reformable with heating) nylon, PVC

Ex. PA 11 (functional prototypes, automotive interior parts, etc.)

- Tensile YS: 41 MPa
- Young's Modulus: 1.3 GPa
- Density: 1.0 g/cm³
- Melting Point: 180 C



Thermoset Plastics: (can't be reformed) silicone, melamine

Ex. Epoxy (adhesives, metal coating, etc.)

- Tensile YS: 14.8 MPa
- Young's Modulus: 2.4 GPa
- Density: 1.3 g/cm³
- Melting Point: 115 C



Elastomers: (elastic) rubbers, neoprene

Ex. Silicone Rubber (food products, footwear, etc.)

- Tensile YS: 10.4 MPa
- Young's Modulus: 16.7 MPa
- Density: 1.2 g/cm³
- Melting Point: N/A, combusts at 410 C



Composites

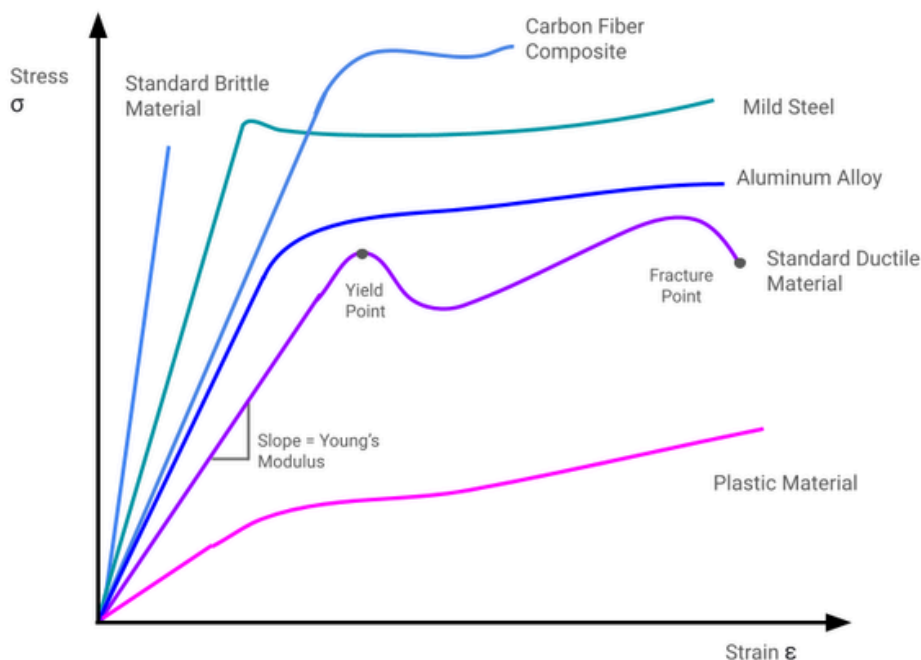
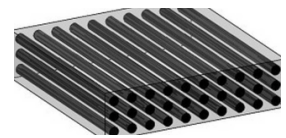
Two or more materials consisting of fibers in a matrix, strong, and light

Fibers: carbon fiber, glass fiber, metallic fibers

Matrices: Metal, ceramic, or polymer matrices

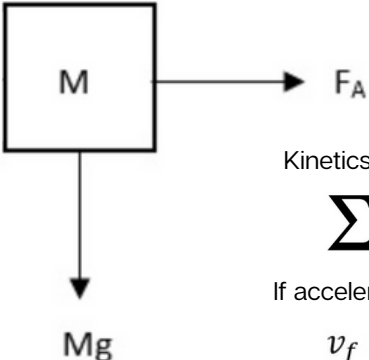
Ex. Carbon Fiber-Reinforced PA 6 (wing spar, body panels)

- Tensile YS: 196 MPa
- Young's Modulus: 23 GPa
- Density: 1.3 g/cm³
- Melting Point: 260 C



Free Body Diagrams + Dynamics

Translational



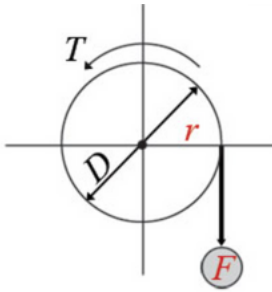
Kinetics to Kinematics:

$$\sum F = ma$$
 If acceleration is constant:

$$v_f = v_i + at$$

$$\Delta x = v_i t + \frac{1}{2} at^2$$
 Kinetic Energy: $E_k = \frac{1}{2} mv^2$ Potential Energy: $U_g = mgh$ Momentum: $p = mv$

Rotational



Kinetics to Kinematics:

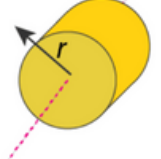


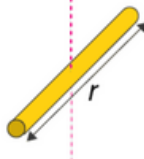
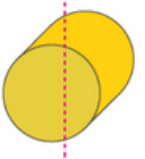


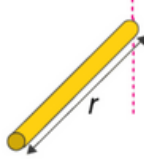
$$\sum \tau = I \alpha$$
 If acceleration is constant:

$$\omega_f = \omega_i + at$$

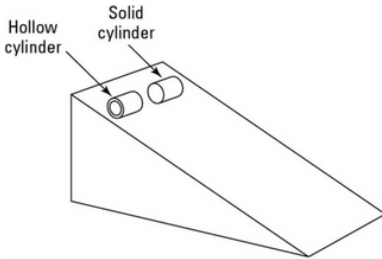
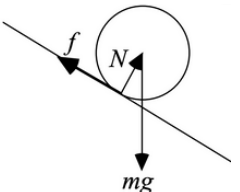
$$\Delta \theta = \frac{1}{2} at^2 + \omega_i t$$
 Kinetic Energy:

$$E_{rotational} = \frac{1}{2} I \omega^2$$
 Momentum:

$$L = I \omega$$
 Moment of Inertia:

Solid cylinder or disc, symmetry axis	Hoop about symmetry axis	Solid sphere	Rod about center
			
$I = \frac{1}{2} MR^2$	$I = MR^2$	$I = \frac{2}{5} MR^2$	$I = \frac{1}{12} ML^2$
$I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$	$I = \frac{1}{2} MR^2$	$I = \frac{2}{3} MR^2$	$I = \frac{1}{3} ML^2$
Solid cylinder central diameter	Hoop about diameter	Thin spherical shell	Rod about end
			

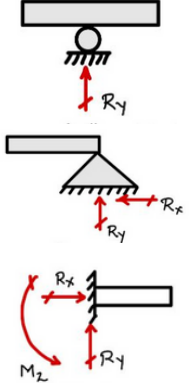
Combined

Rolling Scenario:  Rolling FBD: 

Conservation of Energy:

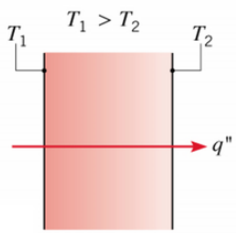
$$mgh = \frac{1}{2} mv_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

Types of Forces + Joints

Normal Force: $F_N = mg$	Lift Force: $F_L = \frac{1}{2} \rho v^2 C_L S$	Buoyancy Force: $F_b = -\rho g V$	
Static + Kinetic Friction: $f = \mu N$	Drag Force: $F_D = \frac{1}{2} \rho v^2 C_D A$		
Spring Force: $F_s = kx$	Centripetal Force: $F_c = \frac{mv^2}{r}$		

Heat Transfer + Fluids

Conduction:



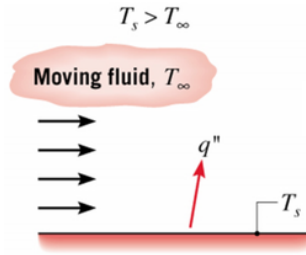
Steady-State:

$$Q = kA \frac{(T_2 - T_1)}{d}$$

Conductivities (W/m-K):

- 316 Steel: 15
- PA 11: 0.28
- Cork: 0.04

Convection:



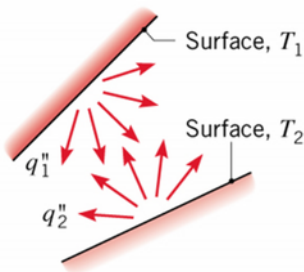
Steady-State:

$$Q = hA(T - T_f)$$

Coefficients (W/m²-K):

- Air (natural): 6-30
- Oil (Forced): 60-18k
- Water (Forced): 300-18k

Radiation:



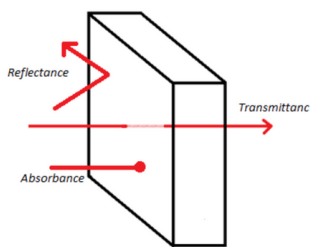
Radiation:

$$E = \epsilon \sigma T_s^4$$

Irradiation:

$$G_{abs} = \alpha G$$

$$\alpha + \rho + \tau = 1$$



Net Radiation ($\epsilon = \alpha$):

$$q_{rad} = \epsilon \sigma A_s (T_s^4 - T_{sur}^4)$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$$

ρ = Reflectivity

α = Absorptivity

τ = Transmissivity

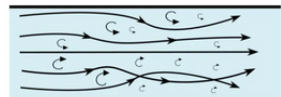
Reynolds Number

$$Re = \frac{\rho u L}{\mu}$$

L = Diameter (Pipes/Cylinders) or Cord Length (Airfoils)

Laminar: $Re < 2300$

Turbulent: $Re > 2300$



Fluids Equations

Volumetric Rate:

$$Q = vA$$

Continuity:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

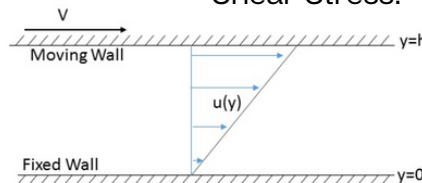
Pressure Variation:

$$P_{gage} = P_{abs} - P_{atm} = \rho g h$$

Bernoulli Equation:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

Shear Stress:



$$\tau = \mu \frac{du}{dy}$$

Pressure Drops in Smooth Pipes:

$$\frac{\Delta p}{L} = f_D \cdot \frac{\rho}{2} \cdot \frac{v^2}{D}$$

Laminar:

$$f_D = \frac{64}{Re}$$

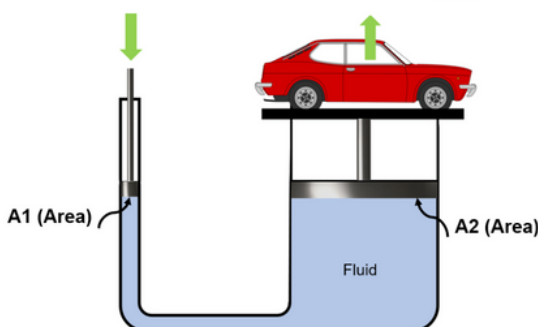
Turbulent:

$$f_D = \frac{0.3164}{Re^{0.25}}$$

Hydraulics + Pneumatics

Pascal's Principle ($P_1 = P_2$):

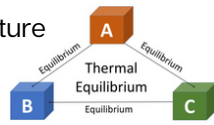
$$F_2 = \frac{A_2}{A_1} F_1$$



Thermodynamics

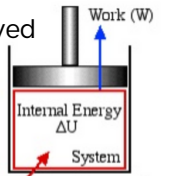
Laws of Thermodynamics

0th: Bodies at the same temperature are in thermal equilibrium



1st: Energy cannot be created or destroyed

$$\Delta U = Q - W$$



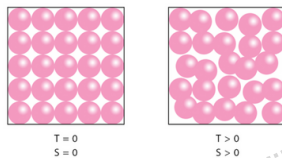
$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m} \left(h + \frac{v^2}{2} + gz \right) = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m} \left(h + \frac{v^2}{2} + gz \right)$$

2nd: Spontaneous processes increase the total entropy of the universe

$$\Delta S_{total} = \Delta S_{system} + \Delta S_{surroundings} > 0$$

e.g. heat moving to colder regions

3rd: A perfectly crystalline compound at zero Kelvin has no disorder and zero entropy



Devices

Nozzles + Diffusers:

Flip application for supersonic flow (e.g. converging/diverging rocket nozzles)

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}$$

Turbines + Pumps + Compressors:

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Heat Exchangers:

$$\dot{m}_1 h_{1,hot} + \dot{m}_2 h_{2,cold} = \dot{m}_1 h_{1,cold} + \dot{m}_2 h_{2,hot}$$

Mixing Chambers:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Psychrometric Definitions

Absolute Humidity: Ratio of water mass to air mass

Relative Humidity: Ratio of water held in air to total possible amount of water that could be held

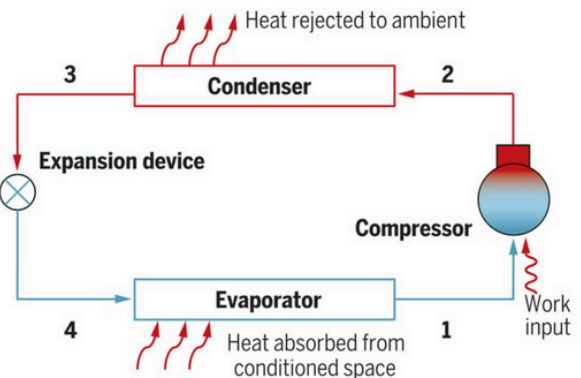
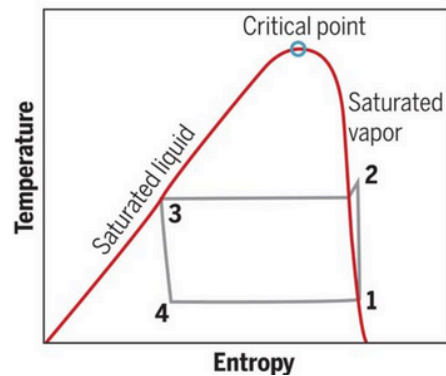
Dew Point Temperature: The temperature at which given air becomes saturated (absolute humidity remains constant)

Wet Bulb Temperature: The temperature at which given air becomes saturated (enthalpy remains constant)

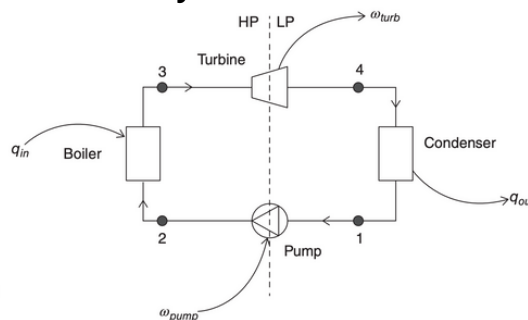
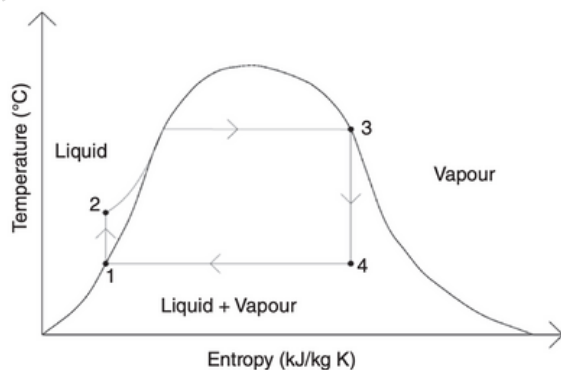
Enthalpy: The heat/energy content of a system

$$H = U + pV$$

Vapor Compression Cycle

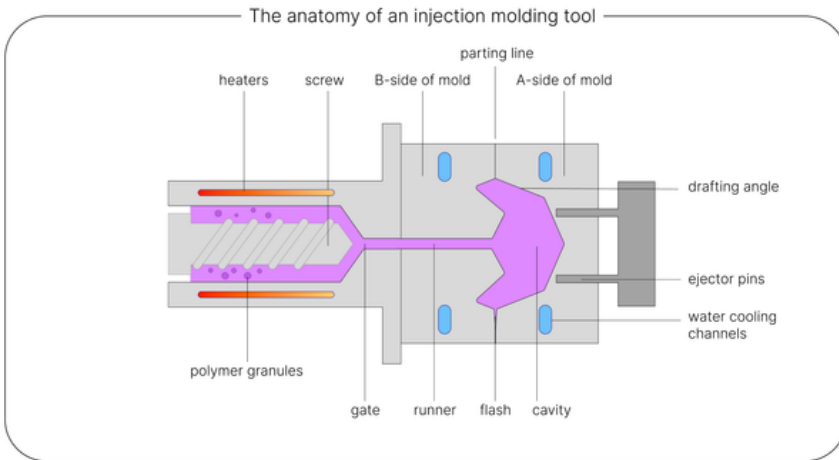


Rankine Cycle



Note: Variations with superheating, staged reheating, and supercritical reheating exist to improve efficiency

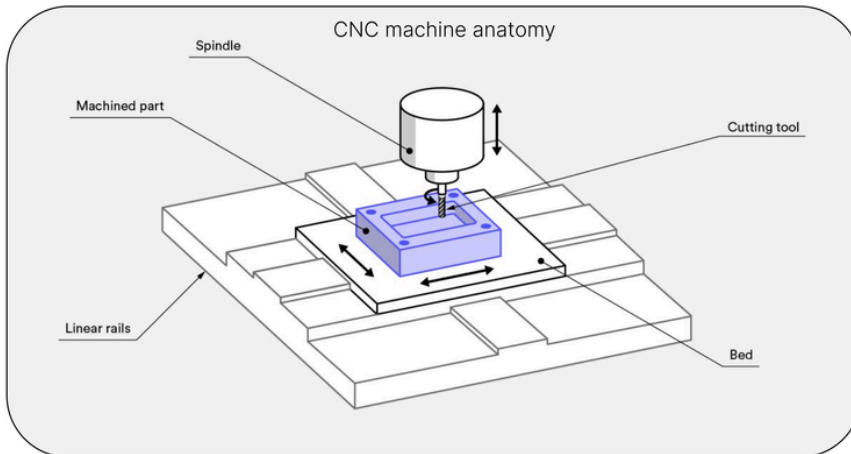
Injection Molding



Plastic Molding Method

- Method of plastic injection where molten plastic is molded under high pressure to produce part in shape of mold cavity.
- **Advantages**
 - High speed of production
 - Low part cost (\$/part)
 - Produce complex geometry with high precision
- **Disadvantages**
 - Long initial lead time for tooling
 - Cost of upfront tooling
 - Lack of design flexibility

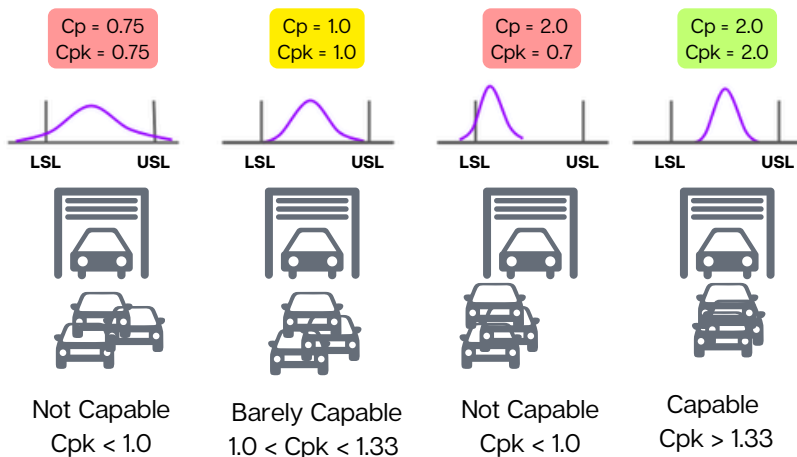
CNC Machining



Subtractive Manufacturing Method

- Material is removed from raw stock material using various cutting tools that rotate at high-speed.
- Software converts 3D model into instructions for CNC machine to generate final part.
- **Material Types**
 - Metals, Plastics, Wood, Foam, Composites, etc.
- **Advantages**
 - Produce parts with high precision.
- **Disadvantages**
 - Unsuitable for high-volume manufacturing
 - High part cost (\$/part)

Statistical Process Control (SPC)



The use of statistical techniques to control a process or production method.

Capability Analysis Metrics:

- **Process Capability (Cp)** measures ability of manufacturing process to produce part within spec. limits.
- **Process Capability Index (Cpk)** measures how close manufacturing process is to center of spec. limits.

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$